



QUESTIONING SKILLS OF PRE-SERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS IN THE PROBLEM-SOLVING PROCESS

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Abstract

Problem-solving has an important place in middle-school mathematics. Mathematics teachers play a significant role in improving problem-solving skills of the students. The purpose of the study is to investigate the questioning skills of pre-service middle-school mathematics teachers in the problem-solving processes of sixth grade students. Fourteen volunteer pre-service middle-school mathematics teachers in the third year of the undergraduate program participated in the study. The participants conducted clinical interviews with one sixth grade student in the scope of provided mathematics problem. The results indicated that most of the participants could not use appropriate questioning considering the phases of problem-solving process. Moreover, these pre-service teachers focused on the solution of the problem and they ignored especially the phases of understanding the problem and developing a plan regarding the problem. Only four of the pre-service teachers were able to support mental process of students in problem-solving via their questioning. The results revealed that enabling students to construct quantitative relationships depends on pre-service teachers' appropriate questioning. Therefore, it is suggested that specific courses developing both the problem-solving and questioning skills of pre-service teachers should be integrated into the mathematics teacher education programs.

Keywords: Teacher Education, Problem Solving, Questioning Skills, Middle School Mathematics.

1. INTRODUCTION

Problem-solving has very important place in mathematics education. Harel (2007: 267) assumes problem-solving not only as a purpose in school mathematics but also as the meaning of mathematics learning. Although getting a correct answer for the problem is praised as being successful in school mathematics, their problem-solving strategies and ways of thinking in the problem-solving process should be focused on (Harel, 2007: 268). Problem-solving has also a notable value in middle-school mathematics. Middle-school mathematics covers the transition from arithmetic to algebra, and in this process the improvement of algebraic reasoning requires to use problem-solving skills (Cai & Knuth, 2011: 62). Problem-solving plays a critical role in overcoming difficulties encountered by students in transition from arithmetic to algebra (Palomares & Hernandez, 2002: 1379). Therefore, problem-solving should be positioned at the center of mathematics education at middle-school level.

The prominent place of problem-solving in mathematics education makes teaching about problem-solving crucial. According to Lester and Lambdin (1999: 41-62) teaching about problem-solving is teaching problem-solving process and/or a number of heuristics rather than teaching mathematics. At this point, Alwarsh (2015: 76) emphasized that the most important thing in teaching about problem-solving is "what students should know and be able to do by focusing on tasks and skills that develop their own thinking and experiences on mathematics" (p. 15). According to Polya (1957: 15) problem-solving is a skill, which can be developed through imitating and practicing such as swimming. While students are solving problems, they observe what their teachers and peers are doing during problem-solving and try to imitate them. Consequently, they start to learn problem-solving while doing it. However, in literature it is stated that subjecting the students only to problem-solving experiences do not support the improvement of their problem-solving skills (Schoenfeld, 1985: 83). Therefore, the teacher has many responsibilities in problem-solving process (Alsawaie, 2003: 41). The most crucial one of these responsibilities is undoubtedly teachers' support for students' thinking skills in problem-solving environment (Polya, 1957: 78). Rigelman (2007: 310) stressed that teachers should actively engage in problem-solving process by asking questions to elicit students' thinking and making sense of multiple approaches. Moreover, the teacher should understand what her/his students think and guide appropriately by asking questions to make the students find the solution by themselves (Polya, 1957: 79). He indicates that the most important points in questioning are leading the

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thinking processes of students to efficient channels and enabling them to choose the appropriate arithmetic operation. By the help of such questioning, the students are able to both ask the questions led by teachers to themselves and carry out the problem-solving process effectively. In this way, their problem-solving and thinking skills may improve.

When the efficient role of teacher questioning in problem-solving process as well as teaching mathematical skills is considered, questioning skills of the teachers come into prominence as one the most important skills of mathematics teachers (Way, 2008: 25). Creating learning environments in which students are encouraged to explain their mathematical ideas, to interpret or to discuss their ideas is closely associated with teachers' ability in questioning (NCTM, 2000: 36). Research studies investigating teachers' questioning skills revealed some important findings with respect to questions to be asked and important points to be considered in questioning (Purdum-Cassidy et al., 2015: 85; Ralph, 1999a: 35; Ralph, 1999b: 290; Way, 2008: 26). For instance, Ralph (1999a; 1999b) emphasized the importance of asking clear and understandable questions stimulating students' thinking as well as providing sufficient time to students to think and give an answer to the question. This point of view puts forward the importance of investigating teachers' questioning skills with respect to how these questions support mathematical thinking of the students.

In the light of related literature and the emphasis on developing pre-service mathematics teachers' questioning skills via mathematics teacher education programs (Moyer & Milewicz, 2002: 293-315), this study aims to investigate questioning skills of pre-service middle-school mathematics teachers in the problem-solving processes of sixth grade students. Within this context, the research question of the study is provided as follows:

- How do pre-service middle-school mathematics teachers' questioning skills in the problem-solving processes of sixth grade students?

It is believed that the results of this research will portray pre-service middle-school mathematics teachers' questioning skills in the problem-solving process of middle-school students who are in the period of transition from arithmetic to algebra.

2. METHODOLOGY

In order to investigate questioning skills of pre-service middle-school mathematics teachers in depth, this study was designed qualitatively.

2.1. Participants

The participants of this study were fourteen (13 females, one male) volunteer pre-service middle-school mathematics teachers enrolled to middle-school mathematics teacher education program. This program consists of general education courses related to teaching profession, general culture courses, mathematics content courses, mathematics pedagogy courses and field experience courses. The participants of the study were seniors and had previously taken all the required courses except some general culture courses like history and philosophy of mathematics and one of the field experience course. The ages of the participants were between 21 and 23, and their grade point averages were between 2.20 and 3.18 out of 4.

2.2. Data Collection

Data for this study was collected through clinical interviews. After the volunteer participants were identified they were informed with respect to clinical interview and its process. The participants were requested to conduct clinical interviews with a sixth-grade student in the scope of provided mathematics problem and record these interviews by audio recorder. At the same time, the participants got permission from the parents of the sixth-grade students they interviewed with. Moreover, participants were required to submit the worksheets used by the students in the problem-solving process to the researchers. It was specifically emphasized that participants were expected to use questioning for understanding and revealing the cognitive processes of the student during the problem-solving process.

2.3. Instrument (A Quantitatively Rich Problem)

In this study, a quantitatively rich problem in the context of real life is used. The problem used to collect in-depth data in clinical interviews is as follows:

Problem: Three friends share 210 Turkish Lira (TL) amount of money. Elif has three times as much money as Ayşe. Levent has 70 TL less than Elif. How much money does Elif get?

This problem leads students to identify quantities and construct relationships among quantities in order to solve the problem. Many studies revealed that quantitatively rich problems enable students to promote problem-solving skills (Ellis, 2007: 450). An instruction, which focuses on calculations and operations and which is deficient in quantitative meaning, may not assist in solving the word problems (Moore, 2011: 301). Moreover, he emphasizes that such an instruction may not support improving in



problem-solving skills and knowledge structures of students. Therefore, mathematics teachers have a critical role to play in encouraging students to identify quantities and construct relationships among quantities in the problem (Ellis, 2007: 452). Weber, Ellis, Kulow and Ozgur (2014: 30) recommend teachers to use practical tips to help students solve the problem and develop quantitative reasoning in the transition from arithmetic to algebra. They state that in the problem-solving process, teachers should “rewrite a problem situation or prompt so that the students must identify the quantities that they believe are relevant to solving the problem”, “ask questions about a problem that focus on why students chose to identify particular quantities and how they intend to or imagine measuring those quantities” and “create natural subparts to a task in which students must articulate their model for a situation and the quantities that constitute it” (Weber et al., 2014, p. 30). In the solving process of this problem, teachers play the role of problem-solving coach by asking students to think about the following questions:

Questions 1: How can you state the data and asked for in the problem?

Questions 2: How can you state the relationships in sharing?

Questions 3: How can you represent the relationships among Elif’s, Ayşe’s and Levent’s money? etc.

By asking students these types of questions, not only can teachers encourage students to identify quantities and construct meaningful relationships among quantities, but they can also support their problem-solving skills and knowledge structures in the problem-solving process (Weber et al., 2014: 30).

2.4. Data Analysis

The texts of the transcriptions of the audio recordings of clinical interviews and participants’ responses to written questionnaire were analyzed by three researchers by using thematic analysis method sparsely and then common themes were determined by working on analysis together. The analysis results of the researchers based on common themes were found to be consistent (Miles & Huberman, 1994: 157).

2.5. Analysis Framework

The analysis framework was constructed by using Polya’s (1957: 26) four-step problem-solving process and practical tips suggested for teachers to help students develop quantitative reasoning (Weber et al., 2014: 30). Polya’s (1957: 26) problem-solving model involves the phases of understanding the problem, developing a plan regarding the problem, carrying out this developed plan and looking back at the solution of the problem. Polya remarks that each problem-solving phase has an importance within itself. At the same time, he stresses that it is encountered with undesired results when any of four phases is skipped or not considered sufficiently. For example, if a student starts to perform an operation or make drawings without understanding the problem, problem-solving would turn out a meaningless process. When the student does not develop a plan, s/he is not able to construct the quantities and the relationships among quantities, s/he is not able to make the main and sub connections in the problem situation as well. In addition, the phase of carrying out the plan is not managed appropriately if the student does not use the mathematical thinking processes efficiently. Besides, if a student does not re-think and evaluate the problem after having solved the problem, s/he may lose some of the best effects (Polya, 1957: 30). Due to all these reasons, all the problem-solving phases affect each other mutually and gain importance within integrity. This constructed framework used to analyze questioning skills of pre-service middle-school mathematics teachers was given in Table 1.

Table 1: Analysis Framework for Questioning Skills of Pre-Service Mathematics Teachers to Support Students’ Quantitative Reasoning in The Problem-Solving Process

Polya’s four-step problem-solving process (1957, ss. xvi-xvii)	Tips for developing quantitative reasoning (Weber et al. 2014, p. 26)	Sample questions
Understanding the problem <i>Students have to understand the problem</i>	<ul style="list-style-type: none"> Rewriting a problem situation or prompt so that the students must identify the quantities that they believe are relevant to solving the problem. Asking questions about a problem that focus on why students chose to identify particular quantities and how they intend to or imagine measuring those quantities. 	<p><i>General</i></p> <ul style="list-style-type: none"> What are the data? What is the condition? Is it possible to satisfy the condition? Can you restate the problem in your own words? Can you think of a picture or a diagram that might help you understand the problem? <p><i>Related to the problem</i></p> <ul style="list-style-type: none"> What are three friends doing? What is the total amount of money? How can you state the sharing among three friends?



		<ul style="list-style-type: none"> • What can you say about Elif's money and Ayşe's money (Elif' and Levent's, Ayşe's and Levent's)? • How can you represent Elif's, Ayşe's and Levent's money?
Devising a plan	<ul style="list-style-type: none"> • Creating tasks in which the students must attend to the measures of the quantities in the problem as they determine relationships between those quantities. • Creating natural subparts to a task in which students must articulate their model for a situation and the quantities that constitute it. Doing so allows the students to reflect on the steps they took to solve the problem and to identify natural points at which to rethink their approach. • Asking students to determine how varying individual quantities affects the rest of the quantities in the model. • Having students develop a representation (physical, visual etc.) of the situation they are modeling that consists of all the quantities and their interrelationships. 	<p><i>General</i></p> <ul style="list-style-type: none"> • Look at the unknown! What do you think to find the unknown? • How can you use the method you suggested? • Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other? • Have you taken into account all essential notions involved in the problem? <p><i>Related to the problem</i></p> <ul style="list-style-type: none"> • What do you think to find Elif's money? • Which plan do you suggest to find Elif's money? • How do you use your plan to represent relationships among Elif's, Ayşe's and Levent's money? • How do you represent Elif's money considering Ayşe's (Levent's) money by using your plan? • Does your plan display all relationships among Elif's, Ayşe's and Levent's money?
Carrying out the plan	<ul style="list-style-type: none"> • Having students test relationships between/among the quantities. • Pushing students to justify why those relationships always or do not always hold. 	<p><i>General</i></p> <ul style="list-style-type: none"> • Can you solve the problem using your plan? • Can you see clearly that your plan of the solution is correct? • Can you prove that it is correct? • If the plan is not working, can you propose a different plan? • Can you check your solution? <p><i>Related to the problem</i></p> <ul style="list-style-type: none"> • Can you find Elif's money by using your plan? • If this is Elif's money you have found how much money Ayşe's (Levent's) has? • How can you be sure that those are Elif's, Ayşe' and Levent's money?
Looking back	<ul style="list-style-type: none"> • Introducing follow-up questions to tasks that create opportunities for the students to revise their solution plans. • Having students revise and retest aspects of their solution plan. 	<p><i>General</i></p> <ul style="list-style-type: none"> • Can you check the result? • Can you derive the result differently? Can you see it at a glance? • Does your solution make sense? • Which methods work? Which methods fail? What you learn from completing this problem? • Can you solve this problem another way? • Can you find an easier way to solve this problem? <p><i>Related to the problem</i></p> <ul style="list-style-type: none"> • Can you find Elif's money by using another way? • Does your solution justify the relationships among Elif's, Ayşe's and Levent's money? • Why do you use this way to find Elif's money?

3. RESULTS

The results obtained from the study indicated that eight of 14 pre-service teachers could not use appropriate questioning with regard to the phases of problem-solving process considering the development



of quantitative reasoning of middle-school students and two of them could hardly ever use questioning. On the other hand, only four of 14 pre-service teachers could use appropriate questioning which supported the student's quantitative reasoning and revealed their cognitive structures during problem-solving process.

It was seen that eight pre-service teachers could not lead the sixth-grade students to identify and interpret the quantities to understand the problem, determine relationships among the quantities to devise and carry out a plan, and finally to examine the solution with respect to quantities. Although some of these sixth-grade students tried to interpret and reason relationships among the quantities and represent the quantities in the problem on their own, pre-service teachers did not support these students' quantitative reasoning process and did not use questioning to reveal their thinking with regard to the quantities in the problem. In other words, these pre-service teachers hindered the quantitative reasoning process of their students who seemed to focus on the quantities. It was seen that these eight pre-service teachers led their students to numbers and to perform arithmetic operations rather than to focus the quantities in the problem. Consequently, they used questioning considering the solution of the problem rather than all the quantities and their interrelationships in the problem.

For instance, it was seen that P1 firstly read the problem and his student began trying to interpret and reason relationships among the quantities in the problem on his own. Although the student began to compare the quantities given in the problem such as Ayşe's and Elif's money and make a conjecture by using "if", P1 did not give his student an opportunity to identify the quantities in the problem and to justify his conjecture. Eventually, it was seen that P1 led his student to solution rather than leading his student to identify the quantities and understand the problem.

P1: (reading the problem) ... What do you understand from this problem?

Student: Three friends are sharing 210 TL. Elif has three times as much money as Ayşe, thus Ayşe has less money than Elif. If Elif has most of the shared money, Levent has 70 TL less than Elif.

P1: What kind of a solution are you thinking about?

P2 and P3 questioned their students about what is given and asked for in the problem. It was seen that students of P2 and P3 responded these questions as given in the problem and tried to represent the quantities by using boxes. At this point, P2 did not use appropriate questioning to make her student identify the quantities and interpret her student's representation of quantities. Instead of this, she led her to solution quickly. On the other hand, P3 gave her student an opportunity to identify and interpret only two of the quantities in the problem and led her to solution quickly although her student began trying to compare and reason relationships among the quantities in the problem on her own.

Student: I want to use boxes. (Drawing boxes on the paper)

P3: How did you represent Elif's money?

Student: By three boxes.

P3: How did you represent Ayşe's money?

Student: By one box.

P3: What do these boxes mean?

Student: Elif has three times as much money as Ayşe. Elif's money is more than Ayşe's. Thus, Ayşe's money is one third of Elif's money. Therefore, Ayşe's money is less than Elif's money.

P3: Well, how do you solve this problem?

P4 questioned the student about what is given and asked for in the problem and the student responded these questions as given in the problem without identifying and interpreting the quantities. Afterwards, P4 led the student to pass quickly to the solution of the problem by asking two questions successively. Although, the student of P4 could not give a response for a while, P4 who expected the student to understand the problem quickly did not lead the student to identify the quantities by asking appropriate questions. Moreover, when the student expressed clearly that he did not understand the problem, P4 made the student read the problem again. Even though, it was clear that the student could not understand and identify the quantities in the problem, P4 stated the problem as making a story of it and led the student to the solution once more.

P4: What does the problem tell you or how can you start solving it?

Student: (Silence) ...

P4: What can you think about and do?

Student: Uh ... I could not understand the problem.

P4: You can read the problem again.



Student: (Reading the problem again) ...

P4: Did you understand the problem?

Student: (Silence) ...

P4: Okay, let's understand it together... (making a story of the problem) ... Now, what can we do in order to find how much Elif has?

Similar to the cases presented above, the rest of the eight pre-service teachers (P5, P6, P7, and P8) also did not use questioning to make the student understand the problem and identify the quantities. Moreover, they led their students to solution quickly.

When the phase of devising a plan was considered, it was seen that none of these eight pre-service teachers used questioning to make their students determine relationships among the quantities, represent and interpret all the quantities and their interrelationships. Consecutively, these pre-service teachers who did not lead their students to find connection between given and unknown to support development of their students' quantitative reasoning skills did not use appropriate questioning related to phase of devising a plan. Moreover, two of these eight pre-service teachers (P6 and P7) neither led their students to think in devising a plan of the solution nor set their students free on devising their own plans. One of these participants expected the student to perform an operation and the other one led the student to the keywords in the problem. For instance, P6 led her student to perform an operation by asking "Can you perform an operation?" after her student stated the given and asked for in the problem. P7 asked the student how he perceived the relationships among the quantities, but ignored whether he was aware of these relations or not. Asking directly the relationships among the quantities hindered her student to explore these relationships on his own. It was seen that although P7 asked her student the relationship existing between quantities, she tried to make her student to interpret these relationships with respect to arithmetic operations via keywords. By this way, P7 ignored her student making sense of quantities given in the problem and neglected to lead her student to represent the quantities and relationships among these quantities step by step. Instead of this, she led her student to focus on keywords and make rote arithmetic operations (e.g. when you see the word "times," you should make multiplication with the numbers you see). It was thought that P7 was not aware with regard to the importance of identifying the quantities and determining relationships among the quantities in the problem-solving process.

P7: According to this problem, what relationship exists between Ayşe's and Elif's money?

Student: Elif has three times more than Ayşe.

P7: What does the word "times" mean to you?

Student: The operation of multiplication.

P7: The operation of multiplication. Well, what relationship exists between Elif's and Levent's money?

Student: Decrease...there is the operation of subtraction.

P7: How do you understand it?

Student: I understand because it says "less".

P7: Well, how can you solve the problem?

Consequently, it was seen that the sixth-grade students of these eight participants tried to find the solution of the problem through performing meaningless arithmetic operations by using the numbers given in the problem in the phase of carrying out the plan. It was thought that since pre-service teachers ignored to lead their students to identify the quantities and determine the relationships among the quantities during questioning process, students focused on the numbers rather than the quantities in the problem. For instance, it was seen that seven of these eight students divided 210 by three after they were led to solution quickly. Few of these students explained the reason for dividing 210 by three by stating, "three friends share 210 TL". In this phase, when the students could not find the solution of the problem by performing meaningless operations, these pre-service teachers tried to lead their students to solve the problem by applying known strategies (e.g. drawing figures or boxes, using models) instead of leading them to propose a different plan. This guidance confused most of the students, who could not identify the quantities and determine relationships among the quantities, and caused them not to reach the solution. Throughout this guidance, it was thought that, several participants tried to make their student to use their own strategies by offering their students to apply strategies that they could not make sense of them. For example, one pre-service teacher (P1) asked the question "can you use variable?" whereas another one (P8) asked the question "can you solve it by using model?" The following excerpt displaying guidance of P8 was given below;

P8: Well, can you solve this by another way? For example, by using model ... by drawing figures.



Student: No...

P8: Can you think of different ways to solve the problem? Can you solve this problem by another way?

Student: No. I guess I would not be able to solve this problem... I do not know any other ways of solution. Probably, I am not able to solve this problem.

P8: Have a go. Don't you think of anything else?

Student: No, I'm thinking about just some complicated things. I am not able to solve this problem.

P8: Now, can you use a model for money in any other way?

Student: In any other way?

P8: Yes. Not the money, but can you use a model for money of Elif, Ayşe and Levent in a different way. Using model means for instance, using boxes instead of money.

Student: Hmm ... (silence)...

P8: Or whatever you want. Can you use different figures?

Considering the last phase that is looking back at the solution of the problem, it was seen that five of these eight participants completely ignored the last phase. On the other hand, other pre-service teachers questioned their students about whether the problem could be solved through other ways. At the same time, only one of these pre-service teachers led the student to check the solution. It was seen that none of these participants stressed the quantities and discussed the relationships among the quantities to support students' both problem-solving and quantitative reasoning skills in this phase.

Thus far, the questioning performances of eight participants were summarized and illustrated considering all the phases of problem-solving process. It was seen that two of other pre-service teachers (P9 and P10) hardly ever led the students or asked the questions to the students in the problem-solving process. One of two participants (P9) left her student on his own throughout the problem-solving process. Moreover, she did not support her student mental process by questioning. Besides, the other participant (P10) completed the problem-solving process by asking short questions requiring the response of "yes/no" or by useless questioning. For instance, P9 did not support her student who began trying to interpret the quantities and reason relationships among the quantities in the problem as it was seen in the following excerpt.

Student: Now we write down Elif, Levent and Ayşe (writing Elif, Levent and Ayşe on the paper separately and underlining them).

P9: Okay. Go on.

Student: Hmm. Elif has three times as much as Ayşe... (silence)

P9: Yes.

Student: Levent has 70 TL less than Elif (writing "less than 70 TL" under Levent's name).

P9: Yes.

Student: Now, if we assign "three times" for Elif's money, we assign "one time" for Ayşe's money. Hmm... (silence). Let's think of this problem by using ratio and proportion.

P9: Okay.

On the other hand, it was seen that the other remaining four participants (P11, P12, P13, and P14) supported the mental processes of their students by leading them to focus on the quantities and relationships among the quantities considering all the phases of problem-solving process. Two of these four participants (P13 and P14) used appropriate questioning beginning from the first phase to the last phase of problem-solving process considering quantities and relationships among the quantities given in the problem. Besides, other two participants (P11 and P12) rapidly passed the phase of understanding the problem without leading their student to identify the quantities. After they realized that their students had difficulty in understanding the problem and devising a plan, they returned at the beginning of the first phase by focusing on the quantities. For example, P11 completely ignored the phase of understanding the problem at the beginning of the interview. After her student stated the problem as it was given, she led her student to solution quickly by saying, "Let's solve the problem". The sixth-grade student who tried to solve the problem by leading of P11 could not find the solution of the problem as the result of meaningless operations she performed by using the numbers without being aware of what these numbers were representing. In this process, P11 realized that the student could not understand the relationships among quantities in the problem. Therefore, P11 changed her way of questioning. She asked the student to leave aside thinking the solution and tried to lead him to focus on the relationships among quantities in the problem. She led her student to determine relationship among the quantities. At this point, her student



began to compare and interpret the quantities by the help of this guidance. After this, she led her student to represent all the quantities and their interrelationships.

P11: Do not think of the solution. Let us consider the relationships among the amounts of money three friends share. What does the sentence "Elif has three times as much money as Ayşe" mean to you?

Student: Elif has more money than Ayşe has. In other words, Elif has three times as much as Ayşe.

P11: Well, how can you show what you are telling now?

Student: For example, let's do it like this. If we assign "one time" for Ayşe's money, we assign "three times" for Elif's money.

P11: How can you write it down or show this?

Student: If we assign one box to Ayşe, then we assign three boxes to Elif.

P11: Okay. Now, let's look at the relationship between Elif and Levent. What is said in this problem about it?

Consequently, P11 supported her student to devise an appropriate plan and carry out this plan successfully by leading him to focus on relationships among the quantities. After he reached the solution, P11 led him to check and justify the solution by providing him an opportunity to use the relationships among the quantities.

Other participant who rapidly passed the phase of understanding the problem and made the student focus on the solution of problem was P12. Although the student told that she was unable to understand this type of problems, P12 ignored the phase of understanding the problem and led her to the solution of the problem in a solution-oriented manner. However, when P12 saw that the student had difficulties in the problem-solving process, she returned at the beginning of the problem-solving process and changed her way of questioning. As in the example of P11, when P12 saw that the student performed meaningless arithmetic operations, she realized that her student could not understand the problem. In this case, she posed two different simpler problem situations including similar relationships among quantities to the main problem. It was seen that P12 gave an opportunity to her student to identify the quantities, determine relationships among the quantities, and check and justify the solutions by interpreting relationships among the quantities in the problem-solving process of these two simpler problems. Subsequently, the student could able to determine relationships among the quantities in these two problems, devise and carry out a plan for them. Finally, P12 led him to the main problem. By this way, her student was able to understand the main problem and complete the problem-solving process successfully considering relationships among all the quantities.

P12: ... okay then let's not think of the solution of the problem. Is it okay if I ask you another problem similar to this problem? (P12 is writing the problem on the paper)

Student: Okay... (reading the problem) ... Ayça and Buse have 40 hairclips in total. If Buse has eight hairclips more than Ayça, how many hairclips each have?

P12: What is given in the problem?

Student: There are 40 hairclips and Buse has eight more hairclips than Ayça.

P12: What is asked for in this problem?

Student: It is asked for how many hairclips each have.

P12: So, we will find the number of hairclips of Buse and Ayça have. What do you think about the solution of the problem?

Student: First, I subtract eight from 40, because Buse has more hairclips than Ayça. Then I divide it by two ... (...performing operation...) The result is 16.

P12: Well, whose number of hairclips is 16?

Student: It is the number of Ayça's hairclips.

P12: Well, is your solution completed?

Student: No, we should find the number of Buse's hairclips. For that, we should add 16 and 8.

P12: Why should we add 16 and eight?

Student: Because Buse has eight more hairclips than Ayça.

P12: Okay. Let's solve another problem with you.

Student: Okay... (Reading the problem) ... The sum of my and my father's ages is 56. If my father's age is three times as much as my age how old is my father?

Similar to P11, it was seen that P12 also led her student to check and justify the solution by providing him an opportunity to use the relationships among the quantities in the phase of looking back.



As it was mentioned, P13 and P14 were the two remaining pre-service teachers who were able to use appropriate questioning by revealing mental process of their students throughout the problem-solving process. It was seen that these participants led their students to identify the quantities in the phase of understanding the problem and to determine relationships among the quantities in the phase of devising a plan. Throughout the interview, they used questioning step by step in order to support and reveal mental processes of the students considering the quantities and relationships among the quantities. For example, P13 led her student to determine relationships among the quantities verbally after he could able to identify the quantities. It was seen that the student stated relationships among the quantities and wanted to solve the problem by using boxes. After this point, P13 led her student to represent relationships among the quantities to devise a plan rather than to solve the problem. When the student represented relationship between two quantities, P13 led him to represent relationships among three quantities.

P13: How can you state the given and asked for in the problem?

Student: Three friends share the money among themselves. Elif has three times as much as Ayşe and Levent has 70 TL less than Elif. Total amount of money and relationships in sharing are given. Elif's money is asked for.

P13: Okay, how can you state the relationships in sharing?

Student: I can write three friends one under the other. Elif has three times as much as Ayşe and Levent has 70 TL less than Elif.... In here, I can use box... I can use a box instead of money. May I start solving?

P13: Not yet. You have just stated that you can use boxes. How can you represent the relationships in sharing by using boxes?

Student: If I assign one box to Ayşe's money, then I should assign three boxes to Elif's money. Levent has 70 TL less than Elif ... (silence)

P13: Well, how can you relate to Elif's money and Levent's money?

Student: ... If Elif is assigned three boxes, then Levent should be assigned three boxes minus 70...

It was seen that the student whose mental process was supported by P13's appropriate questioning could able to reach the solution successfully. It was thought that P13 was aware of the responsibility to improve her student's mental process. She successfully led her student who seemed to have enhanced reasoning skills to think different ways of problem solution in the phase of looking back.

P13: ... for example, in here you use boxes instead of money. What other things can you use instead of boxes for the shared money?

Student: Yes, I can assign other things such as box, line or pencil to money.

P13: Why do you use this way for the solution?

Student: Because it's easier to solve the problem by using boxes.

Differently from the student of P13, the student of P14 displayed that he had tendency to use verbal representation for the quantities in the problem situation. For instance, she represented relationships among the quantities by using the word of "times". In the phase of looking back, P14 questioned her student about whether she could use different representations for the quantities or not. By this way, she tried to lead her to represent relationships among the quantities in a different way.

P14: ... How can you express the relationships other than using "times"?

Student: Instead of "times" I can use different figures such as square or triangle. But I prefer using the word of "times" rather than using boxes.

P14: Well, why did not you try to use any figures to solve the problem?

Student: I prefer writing "times" more. It is easier for me. I generally solve the problems by using and writing words rather than drawing figures.

Moreover, P14 questioned her student about her attitude towards to this problem whereas P13 not only critiqued but also evaluated the problem-solving process together with the student.

4. DISCUSSION AND CONCLUSION

In this research, questioning skills of pre-service middle-school mathematics teachers were investigated in the problem-solving processes of sixth grade students who are in the period of transition from arithmetic to algebra. The results from this study clearly showed that majority of 14 pre-service teachers were not able to use questioning providing support for the mental process of the students in the problem-solving process. Polya (1957: 30) emphasizes that each problem-solving phase is important in itself, all of them affect each other mutually, and therefore it should be considered that all phases gain importance within integrity. He also indicates that it may be possible to encounter undesirable results even when any of



these phases are left out. However, in the current study, most of the pre-service teachers focused on the solution of the problem and they ignored especially the phases of understanding the problem and devising a plan. These pre-service teachers led their students to perform arithmetic operations without leading their students to identify and interpret the quantities and determine relationships among the quantities in the phases of understanding the problem and devising a plan, respectively. When these students who performed meaningless arithmetic operations with the numbers in the problem could not solve the problem in the phase of carry out a plan, the participants led them to solve the problem by using another way again and again rather than leading them to understand the problem and to devise a plan. In this way, it was observed that the participants caused their student to get into a vicious circle in the problem-solving process and not to able to solve the problem. Consequently, it was thought that such guidance hinders students from both solving the problem and using some mathematical skills (e.g., interpreting, reasoning, and comparing) required in the problem-solving process. All the troubles mentioned above emerged as undesired results in the problem-solving process as Polya (1957: 30) indicated. Unfortunately, in the present study, most of the participants did not consider the phase of looking back. In the problem-solving process, the phase of looking back enables the students to reconsider the solution, rethink and examine the ways of solution, and gives students an opportunity to revise their devised plan (Polya, 1957: 30; Weber et al., 2014: 29). For this reason, skipping this phase hinders the problem-solving skills of students to improve (Polya, 1957: 36). Similarly, the results of this study also support the results of Alsawaie (2003: 40), and Harel and Lim (2004: 30) regarding that the mathematics teachers focus on the solution of the problem in learning environment and they ignore developing problem-solving skills of the students. Unfortunately, in this study, only a few participants used questioning step by step in order to reveal students' mental process and managed the problem-solving process successfully considering all the phases of problem-solving process within integrity.

As Polya (1957: 34) indicates, problem-solving skills can be developed through imitating and practicing such as swimming. In the present study, most of the participants used inappropriate questioning such as "Can you perform operation?" "Can you start to solve the problem?". It was thought that it was an unfortunate situation for the students who are expected to be problem solver by taking their teachers as model. Because, this inappropriate guidance may cause students not only to ignore the importance of the quantitative meanings in the problem but also to think problem-solving as performing arithmetic operations by using numbers in the problem and reach the solution as soon as possible. On the other hand, it was thought that one of the pre-service teachers who managed the problem-solving process successfully was an expected model since she did not permit her student to perform an operation without understanding the problem by leading him to identify the quantities in the problem. Besides, being a good model for the students, teachers' responsibilities in the problem-solving process have been frequently emphasized (Alsawaie, 2003: 40). The most important of these responsibilities is considering "what students should know and be able to do by focusing on tasks and skills that develop their own thinking and experiences on mathematics" (Alwash, 2015, p.15). In the present study, it seemed to be that the participants did not aware of these responsibilities. It would be expected that these pre-service teachers had at least some of these responsibilities since they would enter teaching profession in a short span of time and solve the problems together with their students in the classroom.

In the period of transition from arithmetic to algebra, middle-school students' ability to reasoning quantitatively provides productive problem-solving behavior (Smith & Thompson, 2007: 100). Although a quantitatively rich problem was used in this study, most of the participants could not lead their students to solve the problem with respect to quantitative meanings and quantitative relationships. It was thought that these participants ignored the quantities since they focused on the numbers in the problem. Several of these participants did not support quantitative reasoning of their students although these students began to reason quantitatively on their own. Despite the fact that these students could be easily led to reason quantitatively, these pre-service teachers hindered their students' quantitative reasoning by using inappropriate questioning. This result provides further support for Rigelman's (2007: 310) claim that both teachers' purposive questioning to elicit students' desirable thinking in the problem-solving process and teachers' awareness of this responsibility are very important. On the other hand, only a few pre-service teachers focused on the quantities and relationships among the quantities in the problem by asking appropriate questions (e.g., asking questions about identifying the quantities, using different representation for the quantities, etc.). It was thought that these participants considered developing students' quantitative reasoning. Additionally, these two pre-service teachers' leading their students to use different representations for the quantities is especially significant in transition from arithmetic to algebra in respect of contribution to the development of the variable concept in 6th grade students. This is stressed by Smith and



Thompson (2007: 100) who indicate that guiding the students to represent relationships among the quantities rather than to perform calculations in the problem-solving process would be an early route to interpreting the algebraic symbols.

Consequently, it was clear that most of the pre-service teachers could not use appropriate questioning to support their students' development of quantitative reasoning skills in solving process of a quantitatively rich problem. This result confirms view of Ellis (2007: 451) who suggests the solving quantitatively rich problem does not guarantee students' quantitative reasoning efficiently. Additionally, Moore (2011: 303) remarks that a leading which is deficient in respect of quantitative meanings will not assist in both solving word problems and supporting improvement of problem-solving skills. In the current study, it was seen solving the problem without focusing on the quantities and determining relationships among quantities turned to useless problem-solving acting for the students. Therefore, the results of this study clearly revealed the importance of teaching the students to think quantitatively in learning environments (Nathan & Young, 1990: 190). It was thought that all these results provide evidence for that enabling students to identify the quantities and determine relationships among the quantities in the problem depends on teachers who are able to use well-structured questioning (Ellis, 2007: 455).

When the questioning of the participants was investigated in the problem-solving process it was seen that most of them used questioning which were deficient and flawed in stimulating their students reasoning. For instance, most of the pre-service teachers' questions requiring the answers "yes" or "no" such as "Can you perform operation?" "Do you understand?" or "Can you solve the problem?" are regarded as undesirable questions since such type of questions do not support students' higher order thinking skills (Purdum-Cassidy et al., 2015: 85). Way (2008: 26) stressed that questions which are known as "lower order" questions are insufficient in promoting mathematical thinking. On the other hand, the questions of P13 and P14 such as "why do you use this way for the solution?" or "why did not you try to use any figures to solve the problem?" are type of questions to elicit students' reasoning. Some of the pre-service teachers asked questions successively without giving students an opportunity to think whereas some of them asked questions such as "can you use variables?" or "can you solve by modeling?" to students which are not available in their vocabulary. This inappropriate guidance of the pre-service teachers not only hindered mental process of the students but also caused them to be confused. Ralph (1999a: 35; 1999b: 290) emphasizes the importance of asking clear and understandable questions stimulating students' thinking as well as providing sufficient time to students to think and give an answer to the question. Similarly, Polya (1957: 95) indicates that teachers should ask questions from general to specific and in an obvious and understandable way to improve problem-solving skills of students. Moreover, it was seen that some of the participants used questioning by providing more than required information. One of them implied the relationships among the quantities directly and the other one tried to indirectly refer arithmetic operations by leading her student to keywords. Both Rosenshine, Meister and Chapman (1996: 192) and Polya (1957: 96) strongly emphasize that questions including extensive information more than required are not appropriate in problem-solving process. Polya (1957: 96) characterizes such questions as well-intentioned bad questions and indicated that the secret of the problem would be revealed by such questioning. According to him, this questioning is a surprise as if like "a rabbit pulling out of a hat and it is really not instructive" for the students (Polya, 1957, p. 22). On the other hand, if the students do not understand this questioning, it remains as a useless well-intentioned help (Polya, 1957: 23). In line with his ideas; such questioning was seemed to be useless in the current study.

Although some researchers categorized teachers' questions such as lower or higher level, there are researchers emphasizing that the nature of the questions, type and the quality of the communication, and the nature of task should be considered while investigating the quality of teacher questioning (e.g. Carlsen, 1991: 160). In this study, one of the participants posed two simple problems to her student considering the nature of the quantitatively rich problem. It was seen that teacher's leading her student to solve these two simple problems by asking lower level questions was helpful and efficient for her student to understand complex relationships among the quantities in the main problem and to devise an appropriate plan. This situation confirms suggestions of the researchers mentioned above.

In this study, pre-service teachers took the courses of writing skills, speaking skills and mathematics pedagogy courses. When the academic standings of pre-service teachers in these courses were examined, it seemed that there was no interpretable relationship between their academic scores in these courses and their questioning skills. For instance, although the academic scores of all the participants from the courses aiming to develop native language skills were between AA and BA (4 and 3.3 out of 4), the participants showed different performances in using questioning. In a similar way, P4 who took the highest score in the course of



mathematics pedagogy (BB, 3.3 out of 4) did not use appropriate questioning in order to make student to identify the quantities. Moreover, she led the student to solution without evaluating whether she understood the problem or not. On the contrary, P11, who took the lowest score (CB, 2.3 out of 4) in the same course, used appropriate questioning to make her student to reasoning quantitatively, and thus she managed the problem-solving process successfully. The results revealing that most of the pre-service teachers cannot use appropriate questioning and the emphasis on that questioning skills of the mathematics teachers are essentially important in the mathematics education literature put forth the necessity to give pre-service teachers opportunities to develop their questioning skills. Consequently, this situation strengthens the necessity of the specific courses in mathematics teacher education programs in order to improve questioning skills especially in problem-solving (Moyer & Milewicz, 2002: 293-315).

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